

# **INVESTMENT PLANNING**

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## Quantitative Investment Concepts (Topic 35)

### CFP Board Student-Centered Learning Objectives

- (a) Calculate and interpret statistical measures such as mean, standard deviation, z-statistic, correlation, and  $R^2$  and interpret the meaning of skewness, and kurtosis.
- (b) Estimate the expected risk and return using the Capital Asset Pricing Model for securities and portfolios.
- (c) Calculate Modern Portfolio Theory statistics in the assessment of securities and portfolios.
- (d) Explain the use of return distributions in portfolio structuring.
- (e) Identify the pros and cons of, and apply advanced analytic techniques such as forecasting, simulation, sensitivity analysis and stochastic modeling.

#### ***Quantitative Investment Concepts***

- A. *Distribution of returns*
  - 1) *Standard deviation*
  - 2) *Normal distribution*
  - 3) *Lognormal distribution*
  - 4) *Skewness*
  - 5) *Kurtosis*
- B. *Semi-variance*
- C. *Coefficient of variation*
- D. *Combining two or more assets into a portfolio*
  - 1) *Covariance*
  - 2) *Correlation coefficient*
  - 3) *Two-asset portfolio standard deviation*
- E. *Beta*
- F. *Modern portfolio theory (MPT)*
  - 1) *Mean-variance optimization*
  - 2) *Efficient frontier*
  - 3) *Indifference (utility) curves*
- G. *Capital market line*
- H. *Capital asset pricing model (CAPM)*
  - 1) *Security market line*
  - 2) *Limitations of CAPM*
- I. *Arbitrage pricing theory (APT)*
- J. *Other Statistical Measures*
  - 1) *Coefficient of determination ( $R^2$ )*
  - 2) *Z-statistic*
- K. *Probability analysis, including Monte Carlo*
- L. *Stochastic modeling and simulation*

**Variability of Returns**

Risk is the possibility that actual results will be less favorable than anticipated results. The greater is this probability, the greater is the risk. Obviously, then, investment assets whose prices or returns fluctuate widely (percentage wise) over time are more risky than those with less variable prices or returns. Two commonly used measures of a security's variability and volatility are its standard deviation and its beta.

**Distribution of Returns**

Standard deviation is an absolute measure of the variability of results around the average or mean of those results.

**Standard Deviation Calculation**

The standard deviation can be calculated manually, but to save time, you should use a financial calculator. To illustrate, assume that an investment has produced the following results in recent years:

<u>Year</u>	<u>Rate of Return</u>
1	-3.6%
2	7.0%
3	9.0%
4	14.0%
5	-2.2%
6	11.0%

You could compute the mean of these results by adding up the numbers and dividing by 6. You will find it to be 5.87% doing it this way. However, use your financial calculator for both the mean and standard deviation. For example:

On the HP-10B II, press the orange shift key (hereinafter referred to simply as shift), CLΣ, 3.6, +/-, Σ+, 7, Σ+, 9, Σ+, 14, Σ+, 2.2, +/-, Σ+, 11, Σ+, shift, and xy (to get the arithmetic mean of 5.87), shift, σxσy (to get the population standard deviation of 6.56), shift, SxSy (to get the sample standard deviation of 7.19).

The population standard deviation is for the entire series of numbers given. The sample standard deviation is a statistical estimate for a larger universe of numbers of which the numbers given are a subset, such as an historical set of returns. In this course, we will focus primarily on the calculation of the sample standard deviation.

- 
- If you use the HP-12C, press yellow f, CLX, 3.6, CHS, Σ+, 7, Σ+, 9, Σ+, 14, Σ+, 2.2, CHS, Σ+, 11, Σ+, blue g, and  $\bar{x}$  (to get the arithmetic mean of 5.87), blue g, S (to get the sample standard deviation of 7.19). Note

that the HP-12C does not calculate a population standard deviation directly.

On the HP-17B II+, press sum, shift, CLR DATA, Yes, 3.6, +/-, INPUT, 7, INPUT, 9, INPUT, 14, INPUT, 2.2, +/-, INPUT, 11, INPUT, EXIT, CALC, and MEAN (which will give you the answer of 5.87), STDEV (which will give you the answer of 7.19).

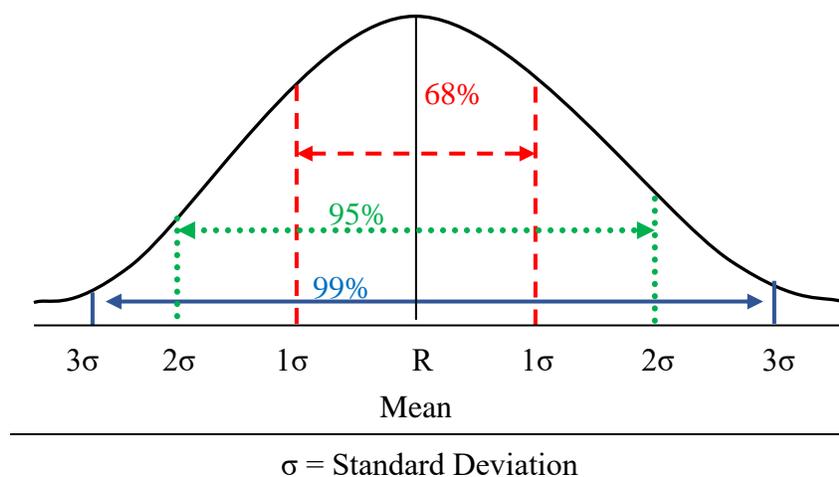
- If you use a BA-II Plus calculator, press 2<sup>nd</sup> data, 2<sup>nd</sup> clear work, 3.6, +/-, enter, ↓↓, 7.0, enter, ↓↓, 9.0, enter, ↓↓, 14, enter, ↓↓, 2.2, +/-, enter, ↓↓, 11.0, enter, ↓↓, 2<sup>nd</sup> stat, 2<sup>nd</sup> clear work, and 2<sup>nd</sup> set (until you see 1 – V on your screen), ↓ (you will see n = 6 on your screen), ↓ (you will see X = 5.87 on your screen), ↓ (you will see Sx = 7.19 on your screen).

### Normal Distributions

In a normal (bell-shaped) distribution, 68% of all results will fall within  $\pm$  one standard deviation of the mean. 95% of all results will fall within two standard deviations of the mean and 99% of all results will fall within three standard deviations of the mean. Likewise, 50% of the results will be higher than the mean and 50% of the results will be lower than the mean.

This diagram shows the normal (bell-shaped) distribution and the standard deviations:

Exhibit 35 – 1



### Lognormal Distributions

A lognormal distribution differs from a normal distribution in that the shape is not necessarily symmetrical. The underlying factors

## Investment Planning – Topic 35

affecting the distribution are, however, normally distributed. For financial planning purposes, a lognormal distribution is used on models when the distribution of certain variables, such as how long clients will live or how much income they will earn, is expected to be skewed.

### **Skewness**

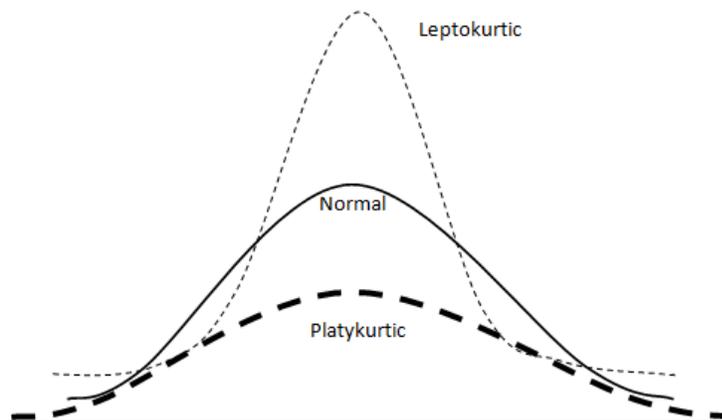
- Skewness measures the symmetry of the bell curve. For example, if the tail to the right of the mean is larger than the tail to the left of the mean, the curve has positive skewness. In a normal bell curve, the two tails are equal, which means the curve has no skewness.

Investors generally are risk averse and will prefer positive skewness to negative skewness because negative skewness means increased downside potential and positive skewness means increased upside potential.

### **Kurtosis**

Kurtosis measures the tallness or flatness of the bell curve. Bell curves with distributions concentrated around the mean and fat tails have a high kurtosis (these are called leptokurtic), while bell curves with evenly spread distributions around the mean and skinny tails have a low kurtosis (called platykurtic). Investors who are risk averse generally prefer low kurtosis to high kurtosis. High kurtosis (with “fat tails”) means that there is increased probability of surprise upside and downside returns such as the “black swan” events mentioned in Topic 34. While the tails are fat on both the upside and the downside, investors tend to react more to the increased downside potential and prefer to avoid such increased downside risk.

**Exhibit 35 – 2**



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### **Semi-variance**

- Semi-variance is a risk measurement of only the downside returns in a portfolio. Only the downside from the mean is averaged since investors seem to fear losses much more than they enjoy positive returns. Morningstar Publications uses this measurement in its star-rating system.

### **Coefficient of Variation**

As discussed at the start of this topic, standard deviation is the absolute measure of variability. The greater the standard deviation, the greater the variability (and, so, riskiness). But, which is more variable, Asset A or Asset B?

Asset A, with an average return of 5.87 and a standard deviation of 7.19

Asset B, with an average return of 6.87 and a standard deviation of 7.59

To answer this, we need a relative measure of variability. That relative measure is the coefficient of variation, which is the standard deviation expressed as a percentage of the mean. In this case, A is riskier because it has a higher relative degree of variability or coefficient of variation.

$$A = 7.19 \div 5.87 = 1.22\%$$

$$B = 7.59 \div 6.87 = 1.10\%$$

### **Practice Question**

Which of the following four investments will provide the least variability and risk?

- A. Investment A: Average return 24%, standard deviation 12
- B. Investment B: Average return 6%, standard deviation 3
- C. Investment C: Average return 12%, standard deviation 5
- D. Investment D: Average return 8%, standard deviation 3

### **Answer:**

The coefficient of variation for these investments is the standard deviation divided by the average return. The investment with the lowest coefficient of variation will be the least variable and least risky. Investment D has the lowest coefficient, with a coefficient of variation of  $3/8 = .375$ .

*The answer is D.*

### **Combining Two or**

- 
- When a portfolio of securities is assembled,

## More Assets into a Portfolio

each of the securities in the portfolio will, of course, have its own standard deviation. The riskiness of the overall portfolio will, therefore, have some relationship to the standard deviation of each of the securities in it, as well as to the proportion of the total portfolio that each security represents within it. However, the riskiness of the portfolio is not simply the weighted average of the standard deviations of the securities in the portfolio. Another factor has to be taken into account, namely, the degree to which the stocks tend to move together. Covariance is one way of measuring this movement. A positive covariance means the stocks tend to move in the same direction, whereas a negative covariance means they tend to move in opposite directions.

### Covariance

- 
- Covariance is the relationship between and among stocks that includes not only the individual stock's variability, but also its effect on and interaction with other portfolio securities. Covariance is the reason the calculation of a portfolio's standard deviation cannot simply be its weighted average.

#### EXHIBIT 35 – 3 Covariance

$$\text{COV.}_{ij} = P_{ij} \sigma_i \sigma_j$$

Where:

$\sigma_i$  = Standard deviation of Asset "i" returns

$\sigma_j$  = Standard deviation of Asset "j" returns

$P_{ij}$  = Correlation coefficient for Assets "i" and "j"

### Correlation Coefficient

The correlation coefficient is another measure of how two assets move in relation to each other. The correlation coefficient measures the relationship between stocks, which is found by dividing the covariance (see below) by the product of the separate standard deviations. The correlation coefficient can be anywhere between +1.0 (perfectly positive correlation) and -1.0 (perfectly negative correlation). A correlation coefficient of 0 means there is

no relationship between the returns for the two investments (they move independently of one another).

**EXHIBIT 35 – 4**  
**Correlation Coefficient**

$$P_{ij} = \frac{\text{COV.}_{ij}}{\sigma_i \times \sigma_j}$$

Where:

$\sigma_i$  = Standard deviation of Asset “i” returns

$\sigma_j$  = Standard deviation of Asset “j” returns

COV. <sub>ij</sub> = Covariance of Assets “i” and “j”

Unless the securities are all perfectly positively correlated (meaning they all move in the same direction at the same time to the same extent), the standard deviation (riskiness) of the portfolio will be less than the weighted-average standard deviation of the individual securities. That reduction in standard deviation is what diversification is all about.

•

**Two-Asset Portfolio  
Standard Deviation**

If a client has a portfolio with just two assets, the standard deviation of that portfolio will not be the average of the standard deviation of the two assets. The reason is not only the weighting of each asset in the portfolio, but also how these two stocks interact together, or their covariance. The standard deviation of the two-asset portfolio will be equal to or less than the weighted standard deviation, depending upon the covariance between the assets.

Unfortunately, there are no shortcuts on the financial calculators to solve for the two-asset portfolio standard deviation. As a result, you will need to use the following formula to solve for the two-asset portfolio standard deviation:

**Two-Asset Portfolio Standard Deviation**

$$\sigma_p = \sqrt{W_i^2 \sigma_i^2 + W_j^2 \sigma_j^2 + 2W_i W_j \text{COV.}_{ij}}$$

Where:

$\sigma_p$  = Standard deviation of the two-asset portfolio

$W_i$  = Weight of Asset “i” in the portfolio

$\sigma_i$  = Standard deviation of Asset “i” returns

$W_j$  = Weight of Asset “j” in the portfolio

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$\sigma_j$  = Standard deviation of Asset “j” returns  
 $COV_{.ij}$  = Covariance of Assets “i” and “j”

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### Example:

What is the standard deviation of the following portfolio?

Stock I

- \$40,000 FMV (40% of portfolio)
- Standard deviation = 2%

Stock J

- \$60,000 FMV (60% of portfolio)
- Standard deviation = 3%

Covariance of Stock I and Stock J = 0.5

The standard deviation of the portfolio is 2.0298% which is calculated as follows:

$$\begin{aligned} & 2 \sqrt{[(0.4^2 \times 2^2) + (0.6^2 \times 3^2) + (2 \times 0.4 \times 0.6 \times 0.5)]} \\ & 2 \sqrt{.64 + 3.24 + .24} \\ & 2 \sqrt{4.12} \\ & = 2.0298\% \end{aligned}$$

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## Beta

The standard deviation of a series of numbers reflects all the **variability** around the mean of those numbers. Consequently, the standard deviation of the returns around the average return reflects all the risks, both systematic (nondiversifiable) and unsystematic (diversifiable), associated with the investment. Beta is quite a different measure of the riskiness of results, one that reflects only the systematic risks, the risks that cannot be diversified away.

**Beta measures a portfolio’s volatility, not variability, relative to some benchmark.** The beta is measuring the impact of this new asset when added to an already-diversified portfolio.



***K Study Tip*** – Note the distinction between the two key terms – *variability* and *volatility*. The former refers to fluctuations around the security’s own mean or average. The latter refers to fluctuations around the market mean or average. Variability is measured by standard deviation; volatility is measured by beta.

**Beta is the measure of risk to use with a diversified portfolio in which unsystematic risk has been removed by diversification.** Standard deviation is the measure of risk to use when a portfolio is not diversified.

 **REMEMBER:** *BETA IS A MEASURE OF SYSTEMATIC RISK; STANDARD DEVIATION IS A MEASURE OF TOTAL RISK (SYSTEMATIC AND UNSYSTEMATIC).*

### Calculation of Beta

Beta is a measure of the **volatility** of a particular security's rate of return or price relative to the volatility of the market as a whole (or the average security in the market). Beta can be calculated using the following formula:

#### **EXHIBIT 35 – 5** **Beta**

$$\beta_i = \frac{P_{im} \sigma_i}{\sigma_m}$$

Where:

$\sigma_i$  = Standard deviation of the individual stock or portfolio

$\sigma_m$  = Standard deviation of the market

$P_{im}$  = Correlation coefficient between the individual stock or portfolio and the market

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#### **Example:**

The standard deviation for a stock is 15%, and the standard deviation for the market is 10%. The correlation coefficient for the stock and the market is 0.6. The beta will be calculated as:

$$\beta = \frac{(0.6 \times 0.15)}{.10} = 0.90$$

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### **Beta as a Measure of Volatility**

The market as a whole has a beta of 1.0, so a security that is just as volatile as the market also has a beta of 1.0. If the betas of securities are more than or less than 1.0, those securities are more or less volatile than the market. For example, Stock A, with a beta

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of 1.3, is 30% more volatile (risky) than the market. Stock B, with a beta of .8, is only 80% as volatile (risky) as the market. Of course, beta is often based on what has happened in the past, so it may not be an accurate indicator of risk in the future.

### **Beta for Mutual Funds**

For a mutual fund, beta is a measure of the fund's volatility in relation to a market index, such as the S&P 500 Index or another index that more closely resembles the fund characteristics. Funds with betas of more than 1.0 are more volatile than the index, while those with betas of less than 1.0 are less volatile than the index.

### **Weighted Beta for Portfolios**

- If a portfolio of securities is assembled that has a weighted-average beta coefficient of 1.0, the variations in the prices and returns should coincide with those of the market as a whole. For example, the following simple portfolio is theoretically a fully diversified one that is exposed only to systematic risk:

<u>Security</u>	<u>Weight</u>	<u>Beta</u>	<u>Wtd. Beta</u>
A	50%	1.3	.65
B	50%	.7	<u>.35</u>
			1.00

### **Practice Question**

An investor's portfolio consists of the following funds:

<u>Fund</u>	<u>Amount</u>	<u>Beta</u>
A	\$45,000	1.10
B	30,000	.90
C	15,000	.75

What is the beta for the investor's portfolio?

- A. 1.00
- B. .975
- C. .917
- D. .900

### **Answer:**

The calculation of the weighted beta is as follows:

$$\text{Fund A: } (45/90) \times 1.1 = .55$$

$$\text{Fund B: } (30/90) \times .9 = .3$$

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$$\text{Fund C: } (15/90) \times .75 = \underline{.125}$$

$$\text{Weighted average} = \underline{.975}$$

The answer is B.

## Modern Portfolio Theory

In his 1952 article entitled “Portfolio Selection,” Harry M. Markowitz laid the foundation for the basic portfolio model. This theory looks at portfolio performance based upon a combination of its assets’ risk and return. With the use of modern computers and the refinement of many complicated quadratic-programming formulas, the theory now provides a framework for helping an investor understand the relationship between risk and return.

The set of portfolios that is the foundation of modern portfolio theory is known as the efficient frontier. Here, the investor attempts to achieve one of the following:

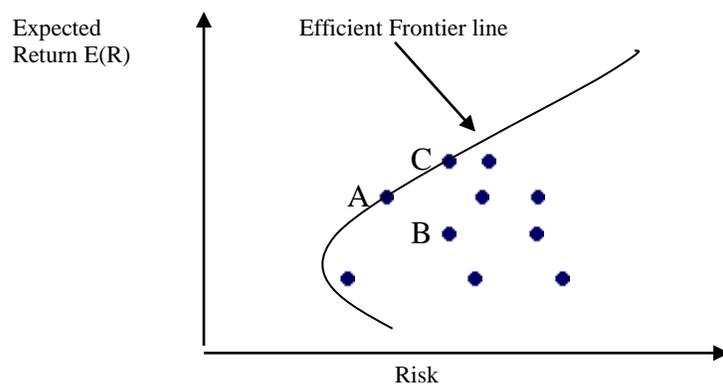
- For a given level of expected return, there is no other portfolio characterized by lower risk.
- For a given level of risk, there is no portfolio with a higher expected return.
- The rational investor will choose the highest expected return with the lowest risk.

## Mean-Variance Optimization

The primary goal of modern portfolio theory is the optimal asset allocation. The mean-variance optimization is a process that can assist investors in the asset allocation by balancing risks and returns.

## Efficient Frontier

Figure 35 – 1



All possible portfolios below the efficient frontier line are attainable, but they may not maximize return for the risk level assumed. **No attainable portfolio can lie above the efficient frontier.** Portfolios outside the efficient frontier are not included in the “feasible set,” or array of possible options. This is because no

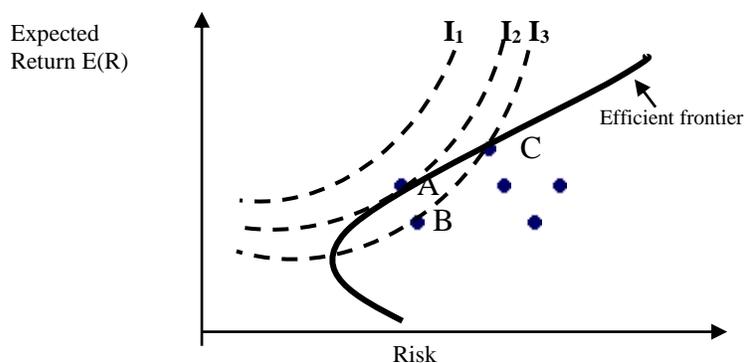
risk and expected return level is attainable at that point. Ultimately, though, it is the individual who will have to decide what combination of risk and return in a portfolio is to be selected. Portfolio A in Figure 35 – 1 is efficient because it lies on the efficient frontier line. For the same level of risk, there is no other portfolio that offers a higher return. Portfolio B is inefficient because for the same level of risk, another portfolio (portfolio C) provides a higher return.

### Indifference (Utility) Curves

Investors attempt to find the optimum portfolio, balancing risk and return by using indifference curves (also called utility curves). These measure the risk/reward *preferences* that an investor is willing to make along the efficient frontier. These curves do not intersect with one another since they represent different investor preferences.

The slope of any curve is a function of the risk-averse nature of the investor. The steeper the slope of the curve, the greater is the investor's aversion to risk. Portfolio possibilities are represented all along the efficient frontier, and the investor is indifferent regarding portfolios that lie on the same indifference curve. For example, portfolios B and C in Figure 35 – 2 below both lie on the same indifference curve ( $I_3$ ), so the investor is indifferent as to these portfolios; he does not perceive one to be of greater value than the other. However, the investor will prefer portfolios on higher curves. The rational investor will seek out the optimum portfolio position. This occurs at a point where the highest indifference curve is tangent to the efficient frontier curve, Portfolio A in Figure 35 – 2. While portfolio C is also on the efficient frontier, this investor prefers portfolio A because it is on a higher indifference curve.

Figure 35 – 2



### The Addition of the Risk-Free Asset

Notice that in Figure 35 – 1, all assets are “risky” assets. In other words, there are no assets on the far left vertical axis where the

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level of risk is zero. It wasn't until 1958 when James Tobin expanded on the work of Markowitz and added the concept of a risk-free asset to the mix. By combining the risk-free asset with a portfolio on the efficient frontier, the ability to leverage the portfolio can create a risk-return profile that is superior to the portfolios on the efficient frontier.

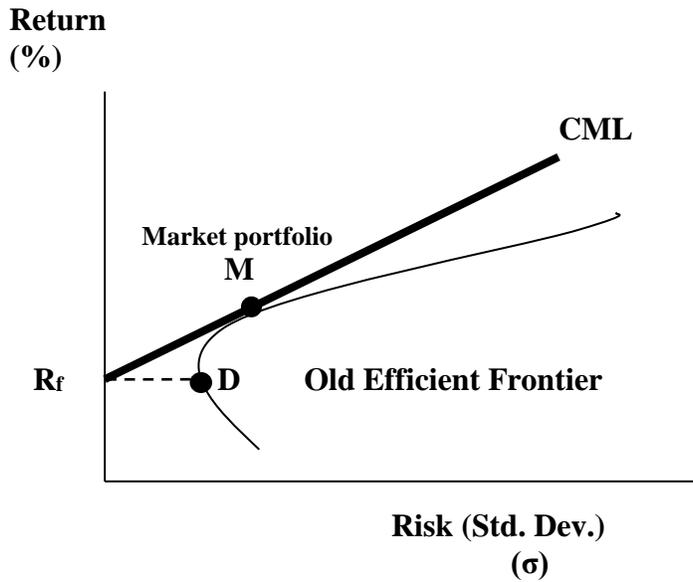
### **Capital Market Line (CML)**

- **The capital market line (CML)** shows the risk-return trade-off for all combinations of risk-free and risky portfolios. For example, in figure 35-3, if all possible combinations of the risk-free asset and portfolio D (e.g. 100%  $R_f$  and 0% D, 80%  $R_f$  and 20% D, 50%  $R_f$  and 50% D, and so on) are plotted, the result will form a straight line between point  $R_f$  and D. However, an investor would not logically choose any combination that includes portfolio D or any other portfolio located to the left of portfolio M, since they are inferior to the returns that can be provided by combining the risk-free asset and portfolio M, which is the market portfolio. The market portfolio M is, conceptually, a portfolio made up of the entire universe of risky assets with the weight of each asset based on its market value. In practicality, the S&P 500 index is often used as a substitute for the market portfolio and U.S. Treasury bills are often used as a proxy for the risk-free asset.
- 
- This new efficient frontier line can now extend to the right of point M and into the area that lies above the old efficient frontier by borrowing at the risk-free rate and using the borrowed funds to invest in the market portfolio.
- 
- The CML is a tool that facilitates the search for the highest expected returns in relation to various levels of portfolio risk, measured by the standard deviation. When depicted graphically, the vertical axis represents the portfolio's rate of return, while the horizontal axis represents the standard deviation of returns, as shown below in

Figure 35 – 3. The CML intercepts the vertical axis of the graph at the rate of return available on a risk-free portfolio,  $R_f$ . The line rises to the right to reflect the fact that for all the available efficient portfolios, return rises as riskiness, measured by standard deviation, rises. The slope of the CML reflects how much more return can be achieved with a given increase in riskiness (or how much more riskiness must be accepted in order to achieve a given increase in return).

The market portfolio point (M) lies where the CML and the efficient frontier line are tangent. To the left of that point is a combination of the risk-free assets and the market portfolio (some assets are invested in the market portfolio, with the remainder being lent at the risk-free rate). To the right of the tangency point is the area in which all of an investor's assets are invested in the market portfolio in addition to using leverage.

Figure 35 – 3



One of the particular goals of the efficient frontier is to select the portfolio with the highest expected return, given a level of risk. The CML model helps investors achieve this goal.

The equation for the CML is:

**EXHIBIT 35 – 6**  
**Required Rate of Return – Capital Market Line**

$$r_p = r_f + \sigma_p \left[ \frac{r_m - r_f}{\sigma_m} \right]$$

Where:

$r_p$  = Expected or required rate of return

$r_f$  = Risk-free rate of return

$r_m$  = Market rate of return

$\sigma_p$  = Standard deviation of the portfolio

$\sigma_m$  = Standard deviation of the market

**Practice Question**

What is the investor's required rate of return on a portfolio using the CML model if the market return is 10% with a 4% standard deviation, the risk-free return is 6%, and the portfolio has a standard deviation of 3%?

- A. 6%
- B. 9%
- C. 12%
- D. 15%

**Answer:**

$$r_p = 6\% + [3\% \times (10\% - 6\%) / 4\%] = 9\%$$

*The answer is B.*

The excess of the market return over the risk-free return ( $r_m - r_f$ ) is known as the market risk premium. In other words, the market risk premium is the extra return provided by investing in the market versus investing in the risk-free asset.

The slope of the capital market line in Figure 35 – 3 is determined by the part of the formula:  $\frac{r_m - r_f}{\sigma_m}$

The slope of the line will change when any of the three variables changes (the return of the market, the risk-free rate, or the standard deviation of the market).

It should be noted that, since CML assumes the only risky asset that is held is the market portfolio, a portfolio made up of all risky assets, it is only useful for diversified portfolios.

**Capital Asset Pricing Model**

The Capital Asset Pricing Model (CAPM) was formulated mostly by William Sharpe (1964) and was built upon the concepts laid down by Markowitz. It relates the risk as measured by beta to the required rate of return or expected level of return on a security and can be used for both diversified and undiversified portfolios (including a single stock). Because of this flexibility, CAPM and the Security market line (SML) is used more frequently than the CML.

Certain assumptions are built into the model and include the following:

Rational investors use like information to formulate the efficient frontier.

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Investors can borrow and lend at the risk-free rate of return.

Taxes, inflation, and transaction costs are equivalent to zero.

There is no preference made for investment decisions on capital gains versus dividend distributions.

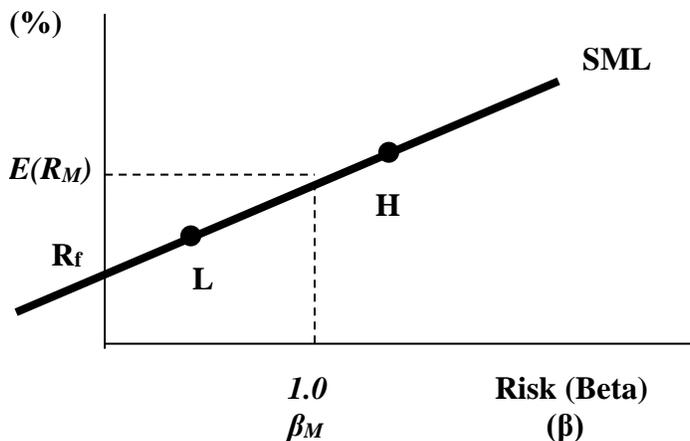
**Security Market Line (SML)**

**CAPM is represented by the security market line (SML).** Here, the slope of the line reflects the risk-return trade-off applicable to individual securities. Though the SML is similar to the CML, there is an important difference. In the SML, riskiness is measured by the beta of the security in question, not by its standard deviation. The beta of the security is based upon that security's impact on the variability of the portfolio.

In the following graph (Figure 35 – 4), point L reflects the choice of a low-risk asset since its return and beta are less than those of the market as a whole. Point H represents the choice of a high-risk asset. Notice that the SML extends to the left of the vertical axis and  $R_f$ . An expected return below the risk-free rate could occur if an asset has a negative beta. Securities with a negative beta would be expected to move in the opposite direction of the market, and would, therefore, perform well if the market were to collapse.

**Figure 35 – 4**

**Return of the Individual Security**



## APPLICATION QUESTIONS

1. (Published question released November, 1994)

The standard deviation of the returns of a portfolio of securities will be \_\_\_\_\_ the weighted average of the standard deviation of returns of the individual component securities.

- A. Equal to
- B. Less than
- C. Greater than
- D. Less than or equal to (depending upon the correlation between securities)
- E. Less than, equal to, or greater than (depending upon the correlation between securities)

2. (Published question released December, 1996)

Which combination of the following statements about investment risk is correct?

- (1) Beta is a measure of systematic, non-diversifiable risk.
- (2) Rational investors will form portfolios and eliminate systematic risk.
- (3) Rational investors will form portfolios and eliminate unsystematic risk.
- (4) Systematic risk is the relevant risk for a well diversified portfolio.
- (5) Beta captures all the risk inherent in an individual security.

- A. (1), (2), and (5) only
- B. (1), (3), and (4) only
- C. (2) and (5) only
- D. (2), (3), and (4) only
- E. (1) and (5) only

3. Stocks X and Y produced the following returns in recent years:

<u>Year</u>	<u>Stock X</u>	<u>Stock Y</u>
1	6%	2%
2	8%	0%
3	4%	10%
4	9%	12%
5	<u>11%</u>	<u>14%</u>
Avg.	7.6%	7.6%

Which of the following are the standard deviations of the returns on the two stocks?

- A. X = 2.7, Y = 6.2
- B. X = 2.7, Y = 4.8
- C. X = 3.8, Y = 6.5
- D. X = 3.8, Y = 5.9
- E. X = 4.1, Y = 5.3

4. Assume that XYZ Corporation's stock has a mean rate of return over the years of 11% and a standard deviation of 3.0. If the historical returns are normally distributed, approximately what percentage of the historical returns have been between 8% and 14%?

- A. 33%
- B. 50%
- C. 68%
- D. 75%
- E. 96%

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5. Which of the following investments is less risky: Stock X, with an average expected return of 20% and a standard deviation of 3; or Stock Y, with an average expected return of 27% and a standard deviation of 5?

- A. Stock X, because it has a lower expected return
- B. Stock X, because it has a lower standard deviation
- C. Stock Y, because it has a higher expected return
- D. Stock Y, because it has a higher standard deviation
- E. Stock X, because it has a lower coefficient of variation
- F. Stock Y, because it has a lower coefficient of variation

6. Assume that a portfolio consists of two stocks, X and Y, and each makes up 50% of the total. Also assume that X and Y have identical standard deviations around their rate of return. In this case, which of the following statements is correct?

- A. If X and Y are perfectly positively correlated, the standard deviation of the portfolio will be twice that of either X or Y.
- B. If X and Y are perfectly negatively correlated, the standard deviation of the portfolio will be zero.
- C. If X and Y are perfectly positively correlated, the standard deviation of the portfolio will be zero.
- D. If X and Y are perfectly negatively correlated, the standard deviation of the portfolio will be twice that of either X or Y.
- E. None of the above conclusions can be drawn because we do not know the market prices of X and Y.

7. The stock of Ajax Corp. has a beta of 1.1, while that of Bohunk Corp. has a beta of .85. In this situation, which of the following statements is correct?

- A. Ajax stock is more volatile than Bohunk stock.
- B. Ajax stock is less volatile than an average stock.
- C. Bohunk stock is more volatile than an average stock.
- D. Both Ajax and Bohunk are less volatile than an average stock.
- E. Both Ajax and Bohunk are more volatile than an average stock.

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For practice answering case questions related to Topic 35, please answer the following questions in the cases included in the Appendix at the back of this textbook.

<b>Case</b>	<b>Questions</b>
Donaldson	5 and 6
Hilbert Stores, Inc.	
Maxwell	
Beals	
Mocsin	
Eldridge	1
Young	
Johnson	6, 7, and 8
Thomas	5, 6, 7, 8, 9, 10, and 11
Jim and Brenda Quinn	12, 13, 14, 15, 16, 17, and 18

## ANSWERS AND EXPLANATIONS

1. **D** is the answer. By combining securities into a portfolio, the standard deviation of the portfolio will, in almost every case, be less than the weighted average of the standard deviations of the individual securities making up the portfolio. The only case that represents an exception to this general principle is if all securities in the portfolio are perfectly positively correlated with each other. As long as there is less than perfectly positive correlation, and especially if there is some degree of negative correlation, the standard deviation of the portfolio will be less than the weighted average of the standard deviations of the individual component securities. Even if there is perfectly positive correlation, the standard deviation of the portfolio will equal, not exceed, the weighted average of the standard deviations of the individual component securities.

2. **B** is the answer. Beta is a measure of systematic, or nondiversifiable, risk. Systematic, or nondiversifiable, risk refers to factors that affect the returns on all similar investments. Therefore, (1) is correct. Since systematic risk cannot be diversified away, by definition, (2) is incorrect. However, since nonsystematic risk can be diversified away, rational investors will form portfolios to do so. Therefore, (3) is correct. In a well diversified portfolio, then, unsystematic risk has been eliminated, so that systematic risk is the only relevant risk. Therefore, (4) is correct. (5) is incorrect because beta captures only systematic, nondiversifiable risk.

3. **A** is the answer. On the HP-10B II, for Stock X, press shift, orange clear all, 6,  $\Sigma^+$ , 8,  $\Sigma^+$ , 4,  $\Sigma^+$ , 9,  $\Sigma^+$ , 11,  $\Sigma^+$ , shift, and SxSy, to produce the answer, 2.7. For stock Y, press shift, orange clear all, 2,  $\Sigma^+$ , 0,  $\Sigma^+$ , 10,  $\Sigma^+$ , 12,  $\Sigma^+$ , 14,  $\Sigma^+$ , shift, and SxSy, to produce the answer, 6.2.

Or, on the HP-12C, for Stock X, press yellow f, CLX, 6,  $\Sigma^+$ , 8,  $\Sigma^+$ , 4,  $\Sigma^+$ , 9,  $\Sigma^+$ , 11,  $\Sigma^+$ , blue g, and S, to produce the answer, 2.7. For Stock Y, press yellow f, CLX, 2,  $\Sigma^+$ , 0,  $\Sigma^+$ , 10,  $\Sigma^+$ , 12,  $\Sigma^+$ , 14,  $\Sigma^+$ , blue g, and S, to produce the answer, 6.2.

4. **C** is the answer. If the historical returns are normally distributed (meaning that they form a symmetrical, bell-shaped curve around the mean), approximately 68% of the returns fell within  $\pm$  one standard deviation of the mean, that is, between 8% and 14%.

5. **E** is the answer. To compare the riskiness of two investments, one must look not at the absolute sizes of the standard deviations, but at the sizes of the standard deviations relative to the average returns; that is, one must compare their coefficients of variation. For Stock X, it is  $3 \div 20$ , or .15. For Stock Y, it is  $5 \div 27$ , or .19. Therefore, Stock X is less risky.

6. **B** is the answer. If the two stocks are perfectly negatively correlated, so that the returns move exactly opposite to each other, the standard deviation of the portfolio is reduced to zero. If they are perfectly positively correlated, on the other hand, the standard deviation of the portfolio will be equal to the average of X and Y.

7. **A** is the answer. A beta of more than 1.0 means the stock is more volatile than average, while a beta of less than 1.0 means it is less volatile. Therefore, A is correct, and B, C, D, and E are incorrect.